Derivation of power formula for several circuits including $N \times N$ Resistors in parallel and series, voltage Divider, current Divider and circuit with Fractal Format

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ABSTRACT

The power formula are obtained for several circuits including $N \times N$ Resistors in parallel and series, voltage Divider, current Divider and circuit with Fractal Format. 2-5

Keywords: Power Formula, $N \times N$ Resistors in parallel and series, Voltage Divider, Current Divider, Fractal circuit.

INTRODUCTION

In previous work 2-5 we obtained the equation for the voltage Divider and current divider for $N$ resistors if we make $N$ resistors equal we obtain the following.

Theory

Resistances in series-parallel. A number of resistors all equal can be connected together in parallel sets if series components, or in series sets of parallel components. By this type if circuits the total power handling capacity can be greatly increased over that of a single resistor.

The overall resistance of a series-parallel network is the same as the value of any one of the resistors if the resistors are identical and are connected in a network called an $n$-by-$n$ matrix.

This means there are $n$ parallel sets if $n$ resistors in series Fig.1 or there are $n$ series sets of $n$ resistors in parallel Fig. 2. The two arrangement eventhough geometrically different produce the same practical result.

Fig. 1 series-parallel configuration
Fig. 2 sets of parallel resistors are connected in series.

The combination of \( n\times n \) resistors will have \( n^2 \) times power capacity of a single resistor. A 3-by-3 series parallel matrix of 2watt resistors can handle \( 3^2 \times 2 = 18 \) watt as an example for obtaining general formula for Fig. 1 and Fig. 2 we start with 1 resistor:

\[
R_{eq} = R \quad \text{and} \quad P_1 = \frac{V^2}{R}
\]

For two by two resistors we obtain

\[
R_{eq} = \frac{R}{2} + \frac{R}{2} = R, \quad P_2 = \frac{V^2}{R}
\]

\[
P_2 = \frac{V^2}{R} + \frac{V^2}{R} + \frac{V^2}{R} = \frac{4V^2}{R} = 4P_1 = 2^2 P_1
\]

For three by three resistors we obtain
For $n$ by $n$ resistors we obtain

\[ R_{eq} = \frac{R}{3} + \frac{R}{3} + \frac{R}{3} = R, P_1 = \frac{V^2}{R} \]

\[ P_n = \frac{9V^2}{R} = 9P_1 = 3^2 P_1 \]

For series parallel configuration we have for 1 resistor:

\[ R_{eq} = \frac{nR}{n} = R, P_1 = \frac{V^2}{R} \]

\[ P_n = \frac{n^2V^2}{R} = n^2 P_1 \]

For series parallel configuration we have for 1 resistor:

\[ R_{eq} = R \text{ and } P_1 = \frac{V^2}{R} \]

For two by two resistors:
For three by three resistors:

\[ R_{eq} = \frac{R}{2} + \frac{R}{2} = \frac{V^2}{R} \]
\[ P_2 = \frac{V^2}{R} + \frac{V^2}{R} + \frac{V^2}{R} = \frac{4V^2}{R} = 4P_1 = 2^2 P_1 \]

For \( n \) by \( n \) resistors

\[ R_{eq} = \frac{R}{3} + \frac{R}{3} + \frac{R}{3} = \frac{V^2}{R} \]
\[ P_3 = \frac{9V^2}{R} = 9P_1 = 3^2 P_1 \]
\[ R_{eq} = \frac{nR}{n} = R, \quad P_1 = \frac{V^2}{R} \]
\[ P_n = \frac{n^2V^2}{R} = n^2P_1 \]

Power loss for voltage divider

\[ P = \frac{V^2}{nR} = \frac{1}{n} \cdot \frac{V^2}{R}, \quad P_n = \frac{V_n^2}{R}, \quad \frac{P_n}{P} = \frac{1}{n} \]

Power loss for current divider

\[ P = nR_{eq}i_n^2, \quad \frac{P_n}{P} = \frac{1}{n} \]

The ratio of power in a branch with respect to total power consumption is obtained to be \( \frac{1}{n} \).

This is interesting that the answers for voltage divider and current divider are the same \( \frac{1}{n} \).

Power consumption by the fractal circuit is as follows:

For 1 resistors

\[ R_{eq} = R \quad \text{and} \quad P_1 = \frac{V^2}{R} \]
For two level of resistors

\[ P_2 = \frac{V^2}{R(1 + \frac{1}{2})}, V = V_1(1 + \frac{1}{2}) \]

\[ P_2 = P_1(1 + \frac{1}{2}) \]

For three level of resistors we have

\[ P_3 = \frac{V^2}{R(1 + \frac{1}{2} + \frac{1}{4})}, V_1 = \frac{RV}{R(1 + \frac{1}{2} + \frac{1}{4})}, V = V_1(1 + \frac{1}{2} + \frac{1}{4}) \]

\[ P_3 = P_1(1 + \frac{1}{2} + \frac{1}{4}) \]
And for \( n \) level of resistors we have

\[
V_1 = \frac{RV}{R(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})}
\]

\[
V = \frac{V_1(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})^2}{R(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})} = P_1(1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})
\]

CONCLUSION

We obtained power consumption and power ratios for several system of circuits for the sets of circuits parallel resistors are connected in series and series parallel configuration voltage divider, current divider, fractal circuit the power ratio and power consumption were obtained.

REFERENCES

[1] Ashourian M., Tabatabaian A., Motalei M., For students of Electrical Engineering Volume 1, publishing Date 1389 Persian Colander Kakas publisher, Isfahan, Iran


