Combination of Trapezoid and Partial Circles

KIUMARS GHOWSI
Islamic Azad University, Department of Chemistry, Majlesi Branch, Isfahan, Iran
HOSEIN GHOWSI
Payame Noor University, Department of Mathematics, Tehran, Iran

ABSTRACT

There are five problems including the trapezoid and partial circles obtained. For the first one the circumference of the black region numerically was obtained. In the problem 2 to 5 the circumference and area of different type of trapezoid and partial circles were obtained. The problems of 4 and 5 are the circumference and the surface area were obtained as function of radius and θ the angle as independent variables. Using trigonometry was helpful.

Keywords: Trigonometry, Trapezoid and circles

INTRODUCTION

When I was tutoring a fifth grader in mathematics, I ran into group of problems involving trapezoid and partial circles. There are five problem concerning trapezoid and partial circles. Problem 1 to 3 requires knowledge of a fifth grader. These problems 1 to 3 are numerical and do not require knowledge of trigonometry. The problems 4 and 5 have been introduced by parametric variables.

PROBLEM 1:

Find the circumference of the black area in the bottom shape.

The answer is: 5+5+(4+3)+(4+3)+4π=36.56

PROBLEM 2:

Find the circumference of black area in the following shape.

The answer is: \[4 + \frac{1}{4}(2\pi 2) + \frac{1}{4}(2\pi 2) = 10.28\]

PROBLEM 3:

Find the surface area of black zone in the following shape.
The answer is: \( 2 \cdot 4 - \frac{1}{4} (\pi \cdot 2^2) - \frac{1}{4} (\pi \cdot 2^2) = 1.72 \)

PROBLEM 4:
Find the circumference of variable black zone in the following shape.

\[
\text{Circumference} = 2\theta r + (2r - 2r\cos\theta)
\]

(\(\theta\) is given by the radian)

For \(\theta = 0\) circumference = 0 (2r - 2r)

For \(\theta = 45 = \pi/8\) circumference = (2\pi r/2) + (2r - 2r\cos\pi/2) = r(\pi + 2)

For \(r = 2\) and \(\theta = \pi/2\) we go back to problem 2 circumference = 2(\pi + 2) = 10.28

This indicates the general formula

\[
\text{Circumference} = 2\theta r + (2r - 2r\cos\theta)
\]

is applicable for the special case of problem 2 where \(r = 2\), \(\theta = \pi/2\).

PROBLEM 5: Find the variable surface area of the black zone in the following shape.

\[
\text{Surface area} = g(r, \theta), \quad S_{\text{trapezoid}} = (l + 2r) \frac{h}{2}
\]
\[ S_1: \]

\[ S_2: \]

\[ S = \pi r^2, \quad S_1 = S_2 = \pi r^2 \theta / 2\pi \]

\[ S_3 \text{ of the black zone:} \quad S_3 = S_{\text{trapezoid}} - (S_1 + S_2) \]

\[ S_3 = (l + 2r) \frac{h}{2} - (2\pi r^2 \theta / 2\pi), \quad h=r\sin\theta, \quad l=2r-2r\cos\theta=2r(1-\cos\theta) \]

\[ S_3 = (2r - 2r \cos \theta + 2r) \frac{r}{2} \sin \theta - r^2 \theta \]

The surface area of the blackzone, \( S_3 \), is after simplification is

\[ S_3 = r^2 (2 - \cos \theta) \sin \theta - r^2 \theta \]

\[ S_3 = r^2 (2 \sin \theta - \cos \theta \sin \theta - \theta) \]

For \( \theta = \pi/2 \), \( S_3 = 2r^2 - \frac{\pi}{2} r^2 \)

For the special case of problem 3, for \( \theta = \pi/2 \) and \( r=2 \) we have the surface area of the black zone=1.72.

This indicates the general formula for the surface area of the black zone is correct.

**CONCLUSION**

There are five problems involving trapezoidal and partial circles both numerically and parametrically were solved. It was shown the first three problems the solution to the problem does not require trigonometry but problems of four and five which are parametrical need trigonometry.

**REFERENCES**
